IMPULSIVE HYDROFRACTURE OF POLYMETHYL METHACRYLATE

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The intensification of geotechnological processes by means of impulse hydrofracture in many areas of the mining industry requires engineering estimates of the parameters of the cracks formed (length, opening, number, orientation), especially if the technology employed requires deliberate fracturing of a solid mass with cracks with definite sizes and configurations. The scheme most widely used for impulsive hydrofracturing consists of the generation of an impulsive pressure in some part of a liquid-filled well. If this pressure is sufficiently high, the circumwell region of the rock mass is fractured with the formation of vertical cracks and radially configured ring-shaped cracks. Since the energy in the pulse and the mass of liquid in the fracture zone are limited, the cracks propagate over a finite distance and have a finite opening, determining the new hydrodynamic properties of the rock mass. In spite of the fact that there exist a substantial arsenal of special geophysical apparatus enabling such work, virtually no attention has been devoted to the mechanics of impulsive hydrofracturing. At the same time, the general characteristics of the fracturing of the rock mass in such loading regimes must be determined in order to predict the effectiveness of impulsive hydrofracturing and to select the optimal conditions. This goal was pursued in the experiments described below.

The rock was modeled with polymethyl methacrylate, since the rupture strength of PMA exceeds the range of variation of this characteristic in real rocks. The sizes of the blocks were chosen so as to guarantee that the crack stops inside the block and does not emerge onto the free surface. An opening (well) 6 (Fig. 1) was drilled in block 5, and a casing tube 7, bounding the region in which the liquid interacts with the wall, was inserted into the opening and glued. A piston 1, equipped with a pressure gauge 8 and channels for injecting the fracturing liquid into working zone 2, reducing air 4, and arranging wires linking the gauge to the recording apparatus 3, was inserted into the tube. The bottom of the opening had a spherical, conical, or flat shape, simulating the type of bits used to drill the wells. The fracturing was performed using three liquids with the same density but substantially different viscosities. The coefficient of kinematic viscosity of the first, second, and third liquids equalled $2.2 \cdot 10^{-6}$, $6.4 \cdot 10^{-4}$, and 0.105 m^2/sec, respectively. The pressure

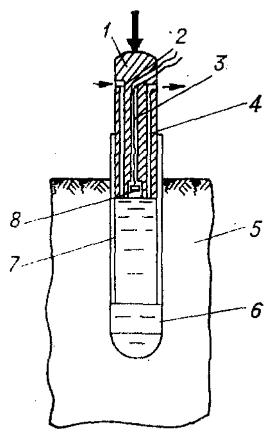

Fig. 1

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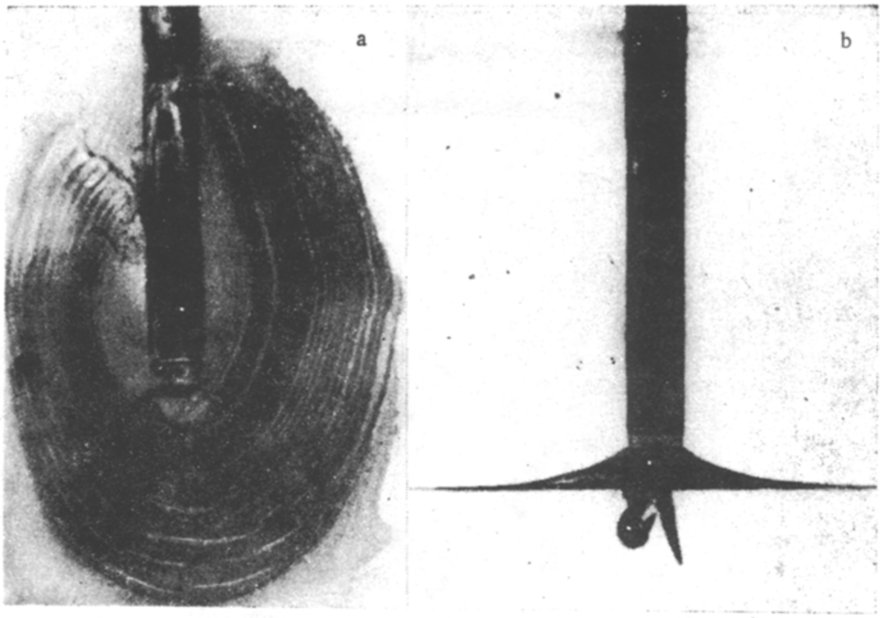

Fig. 2

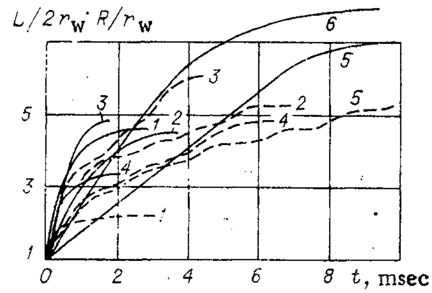

Fig. 3

gauge consisted of a tourmaline or piezoelectric quartz disk, enclosed by a rubber jacket and a metallic holder filled with epoxy resin. The gauge was calibrated under a static load. The matching element in the recording circuit consisted of a single-channel field-effect-transistor amplifier described in [1]. The signals were recorded with an S8-9A oscillograph. The structural elements of the gauge were chosen so that its frequency characteristic would ensure stable recording of the dynamic process with the permissible error. This required that the following two conditions hold [1]:

$$f_{\max} < 0,3 f_g, \quad f_{\min}^2 R_y^2 C_e^2 \gg 1,$$

where f_{max} and f_{min} are the maximum and minimum frequencies of the process being measured; f_g is the characteristic frequency of the gauge; R_y is the input impedance of the amplifier; and C_e is the equivalent capacitance of the measuring channel. In the experiments, $f_{max} \sim$ 2 kHz, $f_{min} \sim 50$ Hz, $f_g = 5 \cdot 10^4$ Hz, $R_y^{-2} C_e^{-2} \leq 0.81$ Hz^2. The random errors in the measurement of the processes associated with the operation of the monitoring—measuring apparatus were determined as in [1]. The total error in the pressure measurements equalled 3.05-16.7% (depending on the amplification regime).

A pressure pulse was excited in the liquid by the impact on the piston of a freely falling 50-kg mass in a 100FU-122 vertical piledriver. The rate of growth of the pressure, which in the experiments varied from $4 \cdot 10^9$ to 10^{11} Pa/sec, was varied by changing the mass of the load, the velocity of the impact, and the thickness of the rubber interlayer between the load and the piston. This range of variation of the growth rate conformed to the possibilities of the well equipment used for impulsive fracture. The temporal development of the cracks was recorded by the optical method using an SKS-1M camera with a speed of 1300-4300 frames/sec, for which the fracturing liquid was colored a dark color. The results of the experiments showed the following.

TABLE 1

No. of curve	v, m/sec	p_m, MPa	$\dot{p}\cdot 10^{-10}$, Pa/sec	$\dot{R}_m$, m/sec	$\dot{L}_m$, msec	t_+, msec	t_g, msec	R/r_w, m	L/r_w, m	$\nu\cdot 10^6$, m/sec
				Radical-ring cracks						
1	1,25	28	1,35	50	—	4,4	1,4	5,0	—	2,2
2	1,98	39	1,96	21	—	6,5	2,0	5,21	—	640
3	3,13	42	2,35	36	—	3,5	1,5	5,05	—	640
4	4,43	53	3,54	40	—	2,9	1,5	3,47	—	640
5	0,77	24	0,66	3,8	—	31,5	3,6	8,67	—	640
6	1,13	26	0,61	10	—	12,6	4,6	8,77	—	$1,05\cdot 10^5$
				Vertical cracks						
1	0,78	22	0,45	—	14	6,6	4,1	—	3,77	2,2
2	0,77	26	0,53	—	29	11,9	4,9	—	13,3	2,2
3	1,40	38,5	1,54	—	35	7,0	2,5	—	11,1	2,2
4	0,77	27,5	0,92	—	14	9,0	3,0	—	8,44	640
5	0,77	20,5	0,40	—	12	18,0	5,0	—	11,1	640

In impulsive hydrofracturing both vertical cracks and radially arranged ring-shaped cracks can form (Fig. 2a, b). The formation of one or another crack depends on the ratio of the height of the fracture zone h to the diameter of the well d_w. For $h > (1\text{-}3)d_w$, vertical cracks are formed. If the fracture zone is smaller, radially arranged ring-shaped cracks are formed. The limiting ratio of h and d_w depends on the configuration of the well bottom. If it is spherical and does not contain any visible stress concentrators, then $h/d_w = 1$. The ratio increases up to 1.8-2.0 for a conical bottom and up to 2.5-3 for a rectangular bottom.

The number of cracks formed depends on the rate of impulsive fracture (the rate of growth of the pressure) and the presence of concentrators. For a pressure growth rate of less than 10^{10} Pa/sec, one crack (bilateral for vertical cracks) usually forms. The number of cracks increases as the fracture rate increases. For low fracture rates the number of cracks can be increased by creating artificial stress concentrators (notches, scratches, etc.). As shown in [2], under production conditions the cracks formed as a result of gun or cumulative perforation of the circumwell zones play the role of such concentrators.

The radial-ring cracks forming in impulsive hydrofracture are horizontal, but in the circumwell zone the plane of the cracks can be curves as a result of the geometry of the concentrator. For example, in wells with a conical or rectangular bottom, the plane of the crack is curved in the direction of the bisector plane of the corner concentrator (see Fig. 2b). This transitional region usually extends over a distance of $(1.5\text{-}2)r_w$, where $r_w = d_w/2$.

The dynamics of the development of the radial-ring and vertical cracks (solid and broken lines) as a function of time is shown in Fig. 3. The conditions of the experiments in which these curves were obtained are given in Table 1, where v is the velocity of the collision of the load and the piston, p_m is the maximum pressure in the liquid; p is the rate of growth of the pressure; $\dot{R}_m$ and $\dot{L}_m$ are the maximum velocities of the radial-ring and vertical cracks, respectively; t_+ is the duration of the pressure pulse; t_g is the growth time of the pressure; R and L are the radius and length of cracks at the moment of stopping; r_w is the radius of the well; and ν is the coefficient of kinematic viscosity.

It is evident from Fig. 3 that at the initial stage of fracture the relation between the instantaneous depth of the cracks and the duration of the pressure pulse is linear. This indicates that the velocity of the cracks is constant. The general character of the curves R(t) and L(t) shows that the crack velocity is at the same time maximum. It should be noted that the absolute values of the maximum rate of growth of the cracks with impulsive fracture of polymethyl methacrylate, achieved in the experiments, is lower by at least an order of magnitude than the maximum possible velocities calculated, for example from the formula of D. K. Roberts and A. A. Williams, namely, $v_T \approx 0.38\sqrt{E/\rho} = 500$ m/sec, where $E = 2.9\cdot 10^9$ Pa is Young's modulus for polymethyl methacrylate and $\rho = 1.18\cdot 10^3$ kg/m^3 is the density. This means that the cracks formed in impulsive hydrofracture are self-stopping; i.e., the energy liberated as the cracks are formed is not high enough for autocatalytic maintenance of the fracture process, which is controlled by the intensity and duration of the load pulse.

Before a crack stops, its velocity decrease monotonically. In addition, the higher the viscosity of the fracture liquid and the lower the rate of growth of the pressure, the

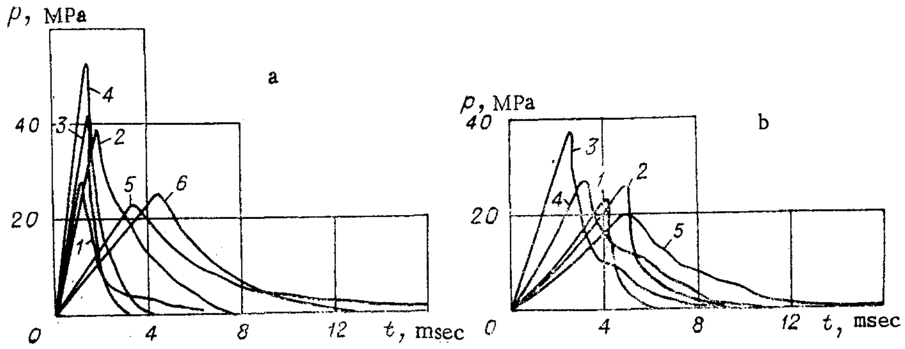

Fig. 4

slower the retardation of the crack is, which, on the whole, increases the duration of the impulse fracture process. The experiments performed established that when the viscosity of the fracture liquid is increased by a factor of 300, the time for development of the crack increases by a factor of three.

Oscillographic records of the change in the pressure as a function of time in the fracture zone are shown in Fig. 4 (a is for radial-ring cracks and b is for vertical cracks; the curves in Figs. 3 and 4 are numbered in the same manner). Comparison of these curves with motion pictures of the motion of the cracks shows that the cracks begin to move at the same time the pressure attains a maximum; i.e., under the conditions of the experiments, the volume of the developing crack increased more rapidly than the volume of the liquid displaced from the well space beneath the piston, and the formation of the crack caused unloading of the medium being fractured. One would expect that there exist rates of loading for which the volume of the liquid flowing out of the well exceeds its rate of absorption by the crack and the onset of motion of the crack will be adapted to some critical pressure, less than its amplitude value. The crack is observed to stop at pressures in the well equal to 5-15% of the pressure existing at the moment of initiation of the crack, which agrees with the general ideas of the mechanics of fracture of materials [3].

Analysis of motion pictures of fracture development reveals that a region free of liquid forms at the tip of the crack; i.e., the front of the crack leads the front of injected liquid. The size of the lead depends on the viscosity of the fracture liquid. Since the velocity of the crack fronts is virtually equal to that of the liquid, the dimensions of this region may be assumed to be constant and to depend only on the properties of the interacting media (solid and liquid).

The characteristic feature of the motion of rupture cracks is its jumplike character (see Fig. 3), which is especially apparent in the stopping sections. This jumplike character is manifested in the form of a periodic alternation of accelerated and retarded development of cracks. The differences in the rates of fracture in these sections are significant: in the section of accelerated development the velocity of a crack reaches 40-60 m/sec, exceeding in many cases the characteristic value for the initial stage of the fracture process, while in the sections with deceleration the crack velocities drop to 1-2 m/sec. The periodicity of the alternation of accelerations and decelerations is quite stable. Its frequency, which depends on the rate of loading and the viscosity of the liquid, varied from 0.67 to 2 kHz. As the rate of growth of the fracture pressure increases, the rate of alternation of the accelerations and decelerations of the cracks increases.

This feature in the development of rupture cracks, established based on the analysis of motion pictures of the process, leads to the appearance on newly formed fracture surfaces of the so-called "stream pattern" [3]. This pattern is most easily observed in relief for low-velocity fracture by a high-viscosity liquid; as the velocity increases, the pattern becomes even finer and may become visually unobservable. The fact that the appearance of the "stream pattern" is linked with the periodic acceleration and deceleration of the cracks is confirmed by measurements of the distances between neighboring crests, which correspond precisely to the frequency of the process. Unfortunately, the method used to measure the pressure pulses precluded the observation of the effect of the periodic alternation of the velocities of cracks on their amplitude–time configuration.

We shall examine the possible mechanism of the periodic alternation of accelerations and decelerations for the example of the fracture of polymethyl methacrylate by a vertical

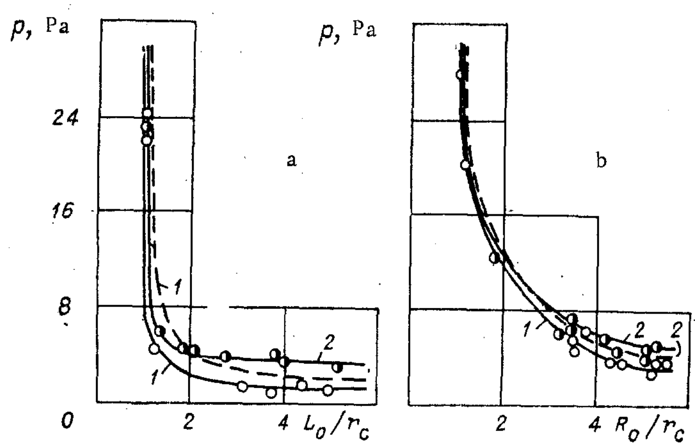

Fig. 5

crack. The motion pictures of the process show that such fracture is initiated most often in the form of a unilateral crack, into which the fracture liquid penetrates at a rate determined by the pressure in it. As the edges of the crack separate under the action of the normally applied pressure, a complicated stress state, created not only by the pressure in the well but also by the strain of the polymethyl methacrylate owing to the appearance of a crack with a nonzero opening, forms around the well. As shown in [4], in this case the highest tensile stresses arise on the opposite (relative to the previously initiated crack) generatrix of the well. When these stresses reach limiting values, a second crack appears in a plane coinciding with the plane of the first crack. The fracture liquid penetrates into this plane, as into the first plane, thereby furthering the development of the fracture process.

Since the flow rate of the liquid determined by the velocity of the piston can be assumed over a sufficiently short time interval to be constant, the appearance of the second crack leads to some reduction of the pressure in the well and to a corresponding deceleration of the cracks. The growth in the pressure owing to the continuing motion of the piston accelerates the rate of growth of the cracks. Fast growth of a crack, however, causes the increase in its volume to lead the volume displaced by the piston, and the pressure in the liquid drops again and the crack slows down, awaiting the next push from the fracture liquid. If this explanation of the periodic acceleration and deceleration of cracks is valid, it becomes obvious that this effect, as well as the formation of the "stream pattern," should be observed most clearly in fracture by vertical cracks and to a lesser extent by radial-ring cracks. This is confirmed by a comparison of the curves in Fig. 3, as well as by the decrease in the period of acceleration as the rate of growth of the pressure in the liquid increases, as mentioned above.

Measurements of the pressure in the fracture zone and its rate of change showed that its threshold value, corresponding to crack initiation, depends strongly on the existence and sizes of initial cracks and other defects on the inner surface of the well (opening). The dependences of the threshold pressure level on the dimensions of the initial crack, shown in Fig. 5 (a is for vertical cracks and b is for radial-ring cracks) are of practical interest. Curves 1 were constructed from the results of experiments with a liquid in which $\nu = 2.2 \cdot 10^{-6}$ m²/sec; for curves 2, $\nu = 6.4 \cdot 10^{-4}$ m²/sec; the broken lines show the theoretical calculations based on the recommendations of [5]. It is evident from Fig. 5 that in order to form radial-ring cracks, under otherwise equal conditions, pressures substantially higher than those required for vertical cracks must be generated in the fracture liquid. This result is in complete agreement with the characteristics of formation of stress fields around an internally pressure-loaded cavity, well known from the theory of elasticity [6]. Thus, for a crack with an initial depth of $2r_w$ the threshold rupture pressure of polymethyl methacrylate for a vertical crack is three to five times lower than for a radial-ring crack. This difference decreases as the initial size of the crack increases and becomes insignificant for $L_0/r_c = R_0/r_c > 5$. Since the threshold rupture pressure decreases rapidly as the dimensions of the starting crack increase, initiation of the starting crack can be used as an effective method for controlling the rupture pressure and the character of the fracture process; this is substantiated theoretically in [2, 5].

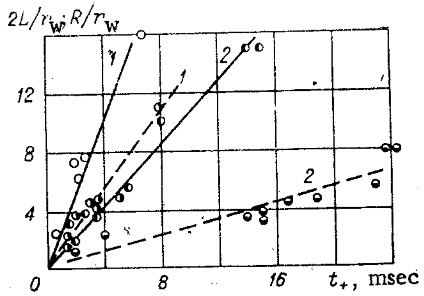

Fig. 6

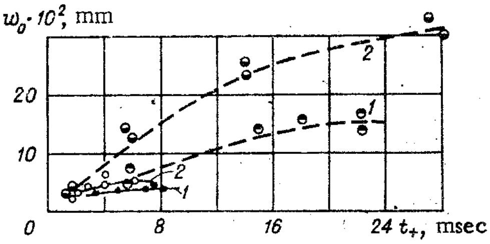

Fig. 7

Since the dimensions of the fracture zone depend on the quantity of liquid injected, it is natural to expect that the depth (length) of the cracks increases as the pulse duration increases. These dependences are shown in Fig. 6 (solid lines for vertical cracks and broken lines for radial-ring cracks). The curves 1 were constructed for fracture with a liquid with $\nu = 2.2 \cdot 10^{-6}$ m²/sec; curves 2 were constructed for a liquid with $\nu = 6.8 \cdot 10^{-4}$ m²/sec. Figure 6 enables drawing two practical conclusions. First, the dimensions of the fracture region increase linearly as the duration of the pressure pulse increases; second, increasing the viscosity of the liquid sharply reduces the dimensions of the fracture zone. Under the conditions of the experiments, increasing the viscosity of the liquid by a factor of 310 reduced the fracture zone by a factor of 2-3.

Since in practice hydrofracture is used to intensify the hydrodynamic interaction of the well and the surrounding medium, some attention should be given to the determination of the opening of cracks, which determines the hydraulic conduction of a crack. In the experiments the irreversible opening of the cracks, determined from the residual strain of the block of polymethyl methacrylate, was recorded. The measurements showed that the maximum opening of the cracks is observed in the walls of the well and can reach 0.2-0.3 mm or $(0.022\text{-}0.033)r_w$. Away from the well, the opening of the cracks decreases, which depends on the type of crack. In vertical cracks the decrease in the opening along the path from the wall of the well to the boundary of the fracture zone is comparatively small and equals 0.1-0.15 mm. In radial-ring cracks the narrowing of the crack in the same region equals 0.15-0.2 mm. It is important to note that near the tip of the crack its opening differs from zero. This suggests that the cracks formed as a result of impulsive fracture exhibit hydraulic conduction in the entire region of their development.

The effect of the duration of the pressure pulse t_+ on the opening of the crack in the wall of the well w_0 is shown in Fig. 7, where the solid lines show the dependences $w_0(t_+)$ for vertical cracks and the broken lines show the same for radial-ring cracks for fracture by liquids with $\nu = 2.2 \cdot 10^{-6}$ and $6.8 \cdot 10^{-4}$ m²/sec (curves 1 and 2). From here, one can draw a conclusion which is of great importance for the technology of impulsive fracture: increasing the viscosity of the fracture liquid leads to growth of the opening of cracks.

Therefore, the parameters of cracks formed with impulsive fracture depend to a significant extent on the viscosity of the liquid. This suggests that the use of properly chosen liquids or their mixtures can be an effective method for controlling the fracture process. As the experiments showed, when a mixture of liquids with a uniform distribution of viscosity over the mass is used for fracture, the crack-propagation process develops according to the laws described above. If immiscible liquids, which become distributed in the well in alternating layers, are used, the fracture of polymethyl methacrylate begins in the region of the liquid with the lowest viscosity, which is injected into the crack first and determines its velocity of propagation and size. Later, the other liquids participate in the development of the process in the order of increasing viscosity, furthering the increase in the opening of the rupture cracks.

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SOME PROPERTIES OF EQUATIONS AND THE METHOD OF THE SMALL PARAMETER IN TWO-DIMENSIONAL SPATIAL PROBLEMS OF THE THEORY OF IDEAL PLASTICITY

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Interest in two-dimensional spatial problems of ideal plasticity theory is due, to a significant degree, to the fact that the majority of industrial operations (rolling, drawing, and immersion of tubes and rods) leads to the study of problems in a three-dimensional coordinate system in which the quantities to be determined depend on two coordinates. We assume that the components σ_{ij} (i, j = 1, 2, 3) of the stress tensor and the components v_i of the velocity vector depend on two variables q_1, q_2 in an orthogonal curvilinear coordinate system q_i ($\sigma_{13} = \sigma_{23} = v_3 = 0$). The Lamé coefficients $H_\ell^2 = \sum_{k=1}^{3}\left(\frac{\partial g_l}{\partial x_k}\right)^2$ are also functions of these coordinates: $H_\ell = H_\ell(q_1, q_2)$. Included in this class are axisymmetric problems (r, z, θ), problems in a spherical coordinate system (r, θ, φ), problems for bodies bounded by coordinate surfaces of degenerate "oblate" and "prolate" ellipsoids, toroidal coordinates, paraboloidal and bipolar coordinates of revolution [1, 2], and many others. The most completely investigated problems are the axisymmetric problems with a Tresca-type plasticity condition [3-5] and some special regimes. Most of the exact and approximate solutions have been obtained for total plasticity [6-10], when the problem becomes locally statically determinate and the system of equations in the stresses and velocities is of hyperbolic type.

The intensive use in recent years of anisotropic powderlike materials, as well as of materials having diverse yield limits in tension, compression, and shear, calls for an analysis of the equations under a more general yield condition. Such an analysis makes it possible not only to extend the class of exact analytical solutions, but also to develop a uniform method for obtaining sufficiently reliable approximate solutions in the event the formulation of exact solutions is not possible. Our aim in the present paper is to analyze some general properties of the equations for two-dimensional spatial problems of plasticity theory and to develop, based on these properties and also extremal properties of limiting loads for rigid-plastic bodies, a uniform method for solving such problems.

1. In the general case involving two-dimensional problems of an ideally rigid-plastic orthotropic body (coordinate axes coinciding with the axes of orthotropy) it is assumed that in the four-dimensional stress space there exists a nonconcave piecewise-smooth yield surface and that there is a valid associated plastic yield law

$$\varepsilon_{ii} = \mu_k \frac{\partial F_k}{\partial \sigma_{ii}}, \quad 2\varepsilon_{12} = \mu_k \frac{\partial F_k}{\partial \sigma_{12}} \tag{1.1}$$

(no summation on i), where

$$\begin{gathered} \mu_k = 0, \ \text{if} \ F_k < 0 \ \text{or} \ F_k = 0, \ dF_k < 0, \\ \mu_k > 0, \ \text{if} \ F_k = 0 \,\text{and}\, dF_k = 0; \\ \varepsilon_{11} = \frac{1}{H_1} v_{1,1} + \frac{v_2}{H_1 H_2} H_{1,2}, \quad \varepsilon_{22} = \frac{1}{H_2} v_{2,2} + \frac{v_1}{H_1 H_2} H_{2,1}, \end{gathered} \tag{1.2}$$

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